Math 156 Applied Honors Calculus II Fall 2018

hw9 , due: Tuesday, November 13

1. The hydrogen atom has one proton and one electron, and according to quantum theory the electron is a cloud of negative charge surrounding the proton. The ground state electron radius is a random variable R with pdf $p(r) = (4/a_0^3)r^2e^{-2r/a_0}$, where $r \geq 0$ is distance from the proton and $a_0 = 5.29 \times 10^{-11}$m is the Bohr radius. (a) Show that $\lim_{r\to\infty} p(r) = 0$. (b) At what radius does $p(r)$ attain its maximum value? Explain why your answer gives a maximum, rather than a minimum or inflection point. (c) Sketch $p(r)$ for $r \geq 0$. (d) Verify that $p(r)$ is a pdf. (e) Find the mean radius of the ground state electron; give the answer in units of a_0. (f) Find the probability that the ground state electron lies within the sphere of radius $4a_0$ centered on the proton.

2. Glucose is introduced into a patient's bloodstream at a constant rate r mg/s. Once in the bloodstream, the glucose is converted to other substances and is depleted at a rate proportional to the amount present. Let $g(t)$ be the amount of glucose in the bloodstream at time t and let k be the depletion rate. Assume $r > 0, k > 0$. a) Write down the differential equation for $g(t)$. b) Let g_0 be the initial amount of glucose in the bloodstream. Find $g(t)$ in terms of g_0, r, k. c) Find $\lim_{t\to\infty} g(t)$. d) Sketch $g(t)$.

3. Determine whether the series converges or diverges; if it converges, find the sum.

a) $3 + 2 + \frac{4}{3} + \frac{8}{9} + \cdots$ b) $\frac{1}{8} - \frac{1}{4} + \frac{1}{2} - 1 + \cdots$ c) $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$

4. Determine whether the series converges or diverges. Justify your answer.

a) $\sum_{n=1}^{\infty} \frac{1}{3n+1}$ b) $\sum_{n=1}^{\infty} \frac{1}{n^3}$ c) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$

5. In each series, how many terms are needed to ensure the error in the nth partial sum, $|s - s_n|$, is less than 10^{-6}? Use the error bounds derived in class. a) $\sum_{n=1}^{\infty} \frac{1}{n^4}$ b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$

6. Consider the series $\sum_{n=1}^{\infty} \frac{x^n}{2^n}$, where x is a variable. (a) For what values of x does the series converge? (b) For those values, find a formula for the sum in terms of x.

7. Given a function $f(x)$ and a point $x = a$, define $T_2(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2}(x - a)^2$; it is a quadratic function of x and it is called the Taylor polynomial of degree 2.

a) Show that $T_2^{(n)}(a) = f^{(n)}(a)$ for $n = 0, 1, 2$, where the superscript indicates the nth derivative. Hence $T_2(x)$ and $f(x)$ agree to 2nd order, and $T_2(x)$ is a quadratic approximation to $f(x)$.

b) Recall from hw8: $f(x) = f(a) + f'(a)(x - a) + \int_a^x (x - t)f''(t)\, dt$.

Now show that $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \int_a^x \frac{(x-t)^2}{2} f'''(t)\, dt$. (hint: in the integral from the hw8 result, set $u = f''(t)$, $dv = (x - t)\, dt$, and integrate by parts)

c) Using part (b), derive the error bound $|f(x) - T_2(x)| \leq \frac{1}{3!} M_3 |x - a|^3$, where $M_3 = \max |f'''(t)|$. (hint: part (b) $\Rightarrow f(x) = T_2(x) + \int_a^x \frac{(x-t)^2}{2} f'''(t)\, dt$; then follow the steps from hw7, problem 10)

d) Let $f(x) = e^x,\ a = 0$. Find $T_2(x)$. Sketch $f(x), T_2(x)$ on the same graph around $x = a$.

e) Make a table with the following format. column 1: $|x - a|$, column 2: $|f(x) - T_2(x)|$. Take $f(x) = e^x, a = 0$ and fill in the entries for $x = 1, 1/2, 1/4, 1/8$ using a calculator. When $|x - a|$ is reduced by a factor of $\frac{1}{2}$, by what factor is the error $|f(x) - T_2(x)|$ reduced?

announcement The 2nd midterm exam is on Wednesday, November 14 at 6:15-7:45pm in 1400 CHEM. If you have a conflict, please tell your instructor. The exam will cover: 1.8 surface area, 1.9 center of mass, 1.10 probability, 2.1 differential equations, 2.2 exponential growth/decay, 2.3 logistic equation, 2.4 Euler's method, 3.1 sequences, 3.2 series, 3.3 convergence tests for series, 3.4 alternating series, hyperbolic functions, Taylor polynomials. A review sheet will be distributed before the exam. Calculators are not allowed on the exam. You may use one sheet of paper (one side) for handwritten notes. We will supply the exam booklets.