

hw5 , due: Friday, October 12

1. The Laplace transform of a function  $f(t)$  is a new function  $F(s) = \int_0^\infty f(t)e^{-st} dt$ ; this construction is used in solving differential equations (Math 216/286/316). Find the Laplace transform  $F(s)$  of the following functions. a)  $f(t) = 1$  b)  $f(t) = e^t$  c)  $f(t) = t$

note : To ensure the integral converges, we assume  $s > 0$  in (a,c) and  $s > 1$  in (b).

2. Consider the integral  $\int_0^\infty \left( \frac{x}{x^2 + 1} - \frac{c}{3x + 1} \right) dx$ , where  $c > 0$  is a constant.

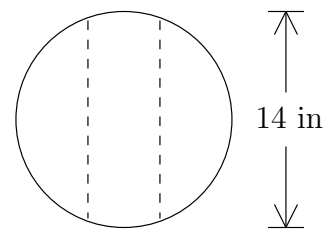
a) Show that evaluating it as  $\int_0^\infty \frac{x}{x^2 + 1} dx - \int_0^\infty \frac{c}{3x + 1} dx$  gives  $\infty - \infty$ , which is undefined.

b) Consider the functions  $\frac{x}{x^2 + 1}$  and  $\frac{c}{3x + 1}$ ; for what value of  $c$  are they asymptotic to each other as  $x \rightarrow \infty$ ?

c) Let  $c$  have the value found in (b), and evaluate the original integral by combining the two antiderivatives. In this way we make sense of  $\infty - \infty$ .

d) The value of the integral obtained in (c) is negative. Explain why that is reasonable.

3. Three students order a 14 inch pizza, and instead of slicing it the usual way, they slice it by two parallel cuts, at  $x = a$  and  $x = -a$ . Find a formula for  $a$  ensuring that each student gets the same amount of pizza. Evaluate the integrals in the formula by the FTC, and solve for  $a$  using Maple (fsolve command) or a calculator. Express the answer in inches.



4. Find the antiderivative by the given method. These antiderivatives were derived in class by other methods; your current answers should be equivalent to those obtained in class.

a)  $\int \sec \theta d\theta = \int \sec \theta \frac{\sec \theta + \tan \theta}{\sec \theta + \tan \theta} d\theta$  , then substitute  $u = \sec \theta + \tan \theta$

b)  $\int \frac{du}{1-u^2} = \int \frac{1-u+u}{1-u^2} du = \int \frac{1-u}{1-u^2} du + \int \frac{u}{1-u^2} du = \int \frac{du}{1+u} + \int \frac{u du}{1-u^2} = \dots$

5. The van der Waals equation of state of a gas gives the pressure  $P$  in terms of the volume  $V$  and temperature  $T$  as  $P = \frac{RT}{V-b} - \frac{a}{V^2}$ , where  $R$  is the ideal gas constant, and  $a, b$  are positive constants depending on the type of molecules in the gas. Note that when  $a = b = 0$ , the vdW formula reduces to the ideal gas law  $PV = RT$ . In an isothermal change of state, the temperature  $T$  is constant, and the work done in compressing the gas from volume  $V_1$  to volume  $V_2$  is  $W = \int_{V_1}^{V_2} P dV$ . Evaluate the integral and find  $W$  in terms of  $V_1, V_2, T, R, a, b$ .

6. Consider a circular sector with radius  $r$  and angle  $\theta$  in the  $xy$ -plane. Let  $L$  be the arclength of the curved sector edge, and let  $A$  be the sector area. Show that  $L = r\theta, A = \frac{1}{2}r^2\theta$ , using the formulas for the arclength of a graph,  $L = \int_a^b \sqrt{1 + (f'(x))^2} dx$ , and the area under a graph,  $A = \int_a^b f(x) dx$ . In each case you need to choose an appropriate  $f(x), a, b$ , and evaluate the formulas to obtain  $L, A$  in terms of  $r, \theta$ . In the case of the area, write  $A = A_1 + A_2$ , where  $A_1$  is the area of a triangle and  $A_2$  is the area under the graph of a curve. (hint: in this problem  $\theta$  is a given fixed parameter; when you apply trig substitution you must use a different symbol for the angle, e.g.  $\phi$ )

