

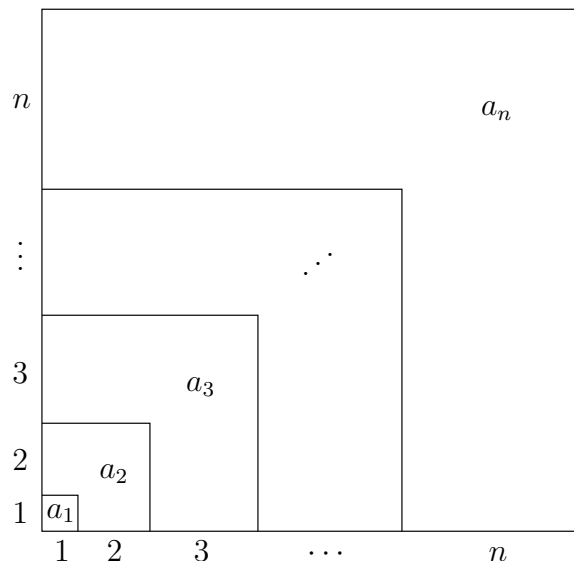
hw2 , due: Tuesday, September 19

1. Show that  $\int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$  using Riemann sums.

2. Show that  $\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$  using two different methods as indicated below.

method a Use a telescoping sum as in class.

method b Consider a square where each side has segments of length  $1, 2, \dots, n$ , so the length of each side is  $1 + 2 + \dots + n = \frac{n(n+1)}{2}$ , and hence the area of the square is  $A = \left(\frac{n(n+1)}{2}\right)^2$ . Now consider subregions of area  $a_1, a_2, \dots, a_n$ , where  $a_1 = 1$  is the area of a unit square, and  $a_i$  for  $i = 2, \dots, n$  is the area of the L-shaped subregion, as shown in the figure. Show that  $a_i = i^3$  for  $i = 2, \dots, n$ , and hence the area of the square is  $A = a_1 + a_2 + a_3 + \dots + a_n = \sum_{i=1}^n i^3$ .



3. Evaluate  $\int_0^1 x^3 dx$  two ways.

a) Riemann sums    b) FTC

4. Evaluate  $\int_a^b x dx$  by Riemann sums.

5. a) Express  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \frac{1}{1+(i/n)^2}$  as an integral. b) Find the derivative of  $f(x) = \int_0^{x^2} \sqrt{1+t^3} dt$ .

6. Evaluate by any method. a)  $\lim_{n \rightarrow \infty} \frac{1}{n} \left( \sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \sqrt{\frac{3}{n}} + \dots + \sqrt{\frac{n}{n}} \right)$     b)  $\int_1^4 \frac{dx}{\sqrt{x}}$

7. A metal rod of length  $L$  m and variable cross-section area  $A(x)$  m<sup>2</sup> has uniform mass density  $\rho$  kg/m<sup>3</sup>, where  $x$  is measured in meters from one end of the rod. (a) Derive an integral expression for the total mass  $M$  of the rod. (hint: think of slices) (b) Compute the total mass for the case  $L = 4$  m,  $A(x) = 9 + 2\sqrt{x}$  m<sup>2</sup>,  $\rho = 1$  kg/m<sup>3</sup>.

8. a) Derive the formula for the sum of a finite geometric series,

$$\sum_{i=0}^n r^i = 1 + r + r^2 + \dots + r^n = \frac{1 - r^{n+1}}{1 - r}, \text{ if } r \neq 1.$$

(hint: check the formula for  $n = 0, 1, 2$ , and then show it is true in general.)

b) A student obtains a \$1,000 loan and repays 50% of the balance each year, i.e. \$500 is repaid in year 1, \$250 is repaid in year 2, and so on. Express the total amount repaid after 10 years as a series and evaluate it using part (a).

c) Evaluate  $\int_0^1 e^x dx$  using Riemann sums. (this completes problem 5c from hw1)

d) What is the value of the sum in part (a) if  $r = 1$ ?

9. Consider the integral  $I = \int_0^1 e^{-x} dx = 1 - e^{-1} = 0.63212056$ . Let  $R_n, M_n$  be the right-hand and midpoint Riemann sums with  $n$  intervals. Construct a table as follows (use a calculator). column 1:  $n$  (take  $n = 1, 2, 4$ ); column 2:  $\Delta x$ ; column 3:  $R_n$ ; column 4:  $|I - R_n|$ ; column 5:  $M_n$ ; column 6:  $|I - M_n|$ . For a given value of  $n$ , which method gives a more accurate answer? When  $\Delta x$  decreases by  $1/2$ , by what factor does the error decrease for each method?

10. Sketch the graph of  $f(x)$  on the given interval. Label the axes.

a)  $f(x) = e^x, -\infty < x < \infty$     b)  $f(x) = \ln x, 0 < x < \infty$

announcement The Science Learning Center ([www.lsa.umich.edu/slc](http://www.lsa.umich.edu/slc)) offers study groups for Math 156 students. Online registration begins on Wednesday Sept 13 at 12pm and groups start meeting on Sunday Sept 17. If the group you want is filled, join the waitlist and another group may be opened.