

hw1 , due: Tuesday, September 11

Please write the solutions neatly (copy over if needed) and staple the sheets together.

1. Write the sum in sigma notation.

a)  $1 + 3 + 5 + 7 + \cdots + (2n - 1)$

b)  $1 + 2 + 4 + 8 + 16 + 32$

c)  $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36}$

d)  $x + x^2 + x^3 + \cdots + x^n$

e)  $1 - x + x^2 - x^3 + \cdots + (-1)^n x^n$

2. True or False? Give a reason or a counterexample to justify your answer.

a)  $(f(x)g(x))' = f'(x)g'(x)$

b)  $\sum_{i=0}^n (n - i) = \sum_{i=0}^n i$

3. Evaluate the telescoping sum.

a)  $\sum_{i=1}^5 ((i + 1)^3 - i^3)$       b)  $\sum_{i=1}^n ((i + 1)^3 - i^3)$

4. Prove the following results.

a)  $\sum_{i=1}^n (a_i + b_i) = \sum_{i=1}^n a_i + \sum_{i=1}^n b_i$

b)  $\sum_{i=1}^n i^2 = \frac{n(n + 1)(2n + 1)}{6}$  (hint: start from  $(i + 1)^3 - i^3 = \dots$ , follow example from class)

5. Each set below defines a region in the  $xy$ -plane. Sketch the region, express the area as a limit of Riemann sums, and evaluate the limit.

a)  $\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1 + x\}$

b)  $\{(x, y) : -1 \leq x \leq 1, 0 \leq y \leq 1 - x^2\}$

c)  $\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq e^x\}$

(note: if you can't evaluate the limit in (c), skip it for now - this requires the formula for the sum of a finite geometric series - you will learn that on hw2.)

6. The area of a certain region  $\{(x, y) : a \leq x \leq b, 0 \leq y \leq f(x)\}$  in the  $xy$ -plane is given as a limit of Riemann sums,  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(5 + \frac{2i}{n}\right)^3 \frac{2}{n}$ . If  $a = 5$ , find  $b$  and  $f(x)$ . Is there another choice of  $a, b, f(x)$  that works?7. Sketch the graph of  $f(x)$  on the given interval. Label the axes.

a)  $f(x) = \sin x, 0 \leq x \leq 2\pi$       b)  $f(x) = \sin 2x, 0 \leq x \leq 2\pi$