Math 156 Applied Honors Calculus II Fall 2017

hw1 , due: Tuesday, September 12

Please write the solutions neatly (copy over if needed) and staple the sheets together.

1. Write the sum in sigma notation.

a) $1+3+5+7+\cdots+(2n-1)$

b) $1+2+4+8+16+32$

c) $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\frac{1}{36}$

d) $x+x^2+x^3+\cdots+x^n$

e) $1-x+x^2-x^3+\cdots+(-1)^n x^n$

2. True or False? Give a reason or a counterexample to justify your answer.

a) $(f(x)g(x))' = f'(x)g'(x)$

b) $\sum_{i=0}^{n}(n-i) = \sum_{i=0}^{n} i$

3. Evaluate the telescoping sum.

a) $\sum_{i=1}^{5}((i+1)^3 - i^3)$ b) $\sum_{i=1}^{n}((i+1)^3 - i^3)$

4. Prove the following results.

a) $\sum_{i=1}^{n}(a_i+b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$

b) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ (hint: start from $(i+1)^3 - i^3 = \ldots$, follow example from class)

5. Each set below defines a region in the xy-plane. Sketch the region, express the area as a limit of Riemann sums, and evaluate the limit.

a) $\{(x,y) : 0 \le x \le 1,\ 0 \le y \le 1+x\}$

b) $\{(x,y) : -1 \le x \le 1,\ 0 \le y \le 1-x^2\}$

c) $\{(x,y) : 0 \le x \le 1,\ 0 \le y \le e^x\}$

(note: if you can't evaluate the limit in (c), skip it for now - this requires the formula for the sum of a finite geometric series - you will learn that on hw2.)

6. The quantity $\lim_{n\to\infty} \sum_{i=1}^{n} \left(5 + \frac{2i}{n}\right)^3 \frac{2}{n}$ is the area of a region $\{(x,y) : a \le x \le b, 0 \le y \le f(x)\}$ in the xy-plane. Find a, b and $f(x)$. Is the choice of a, b and $f(x)$ unique?

7. Sketch the graph of $f(x)$ on the given interval. Label the axes.

a) $f(x) = \sin x\,,\ 0 \le x \le 2\pi$ b) $f(x) = \sin 2x\,,\ 0 \le x \le 2\pi$